Order of the QCD transition and QCD sum rules

C. Adami and G. E. Brown
Physics Department, State University of New York at Stony Brook, Stony Brook, New York 11794
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We propose using the finite-temperature ρ-meson mass as an order parameter to monitor the QCD transition. This is suggested by the ρ-meson mass formula that emerges from finite-temperature QCD sum rules in the vector channel, and which encompasses the effects of both quark and gluon condensates. We find that a second-order chiral-restoring transition implies a second-order behavior for the ρ mass even if the value of the gluon condensate is unaffected by the transition.

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In this Brief Report we would like to explore the implications of chiral restoration on the order of the QCD transition. Historically, the order of the chiral-symmetry-restoration transition was believed to reflect the order of the QCD transition, more so as it was believed that chiral restoration is accompanied by deconfinement. However, there is now ample evidence to the contrary and that the QCD transition to a new phase signaled by the onset of deconfinement is accomplished only at temperatures considerably higher than the chiral-symmetry-restoring temperature T_χ [1]. This is also reflected in the observation of hadronic modes above T_χ [2].

It is believed that the lack of deconfinement at temperatures $T\sim T_\chi$ is due to the persistence of magnetic interactions between quarks, reflected by an area-law behavior of the Wilson loop [3]. The area-law behavior can be traced back to a persistence of the QCD string tension (magnetic-flux lines) above T_χ, while electric interactions are screened. Thus, as confirmed by lattice QCD calculations [4,5], the magnetic gluon condensate $\langle \mathbf{B}^2\rangle$ seems to persist above T_χ, preventing deconfinement. Consequently, the quark condensate may have only limited usefulness as a global order parameter for the QCD transition from hadrons to a plasma of deconfined constituents.

Of course, the choice of an order parameter is merely a matter of convenience. We would like to suggest here using the mass of the lightest vector excitation of the system, as QCD sum-rule formulas imply that the ρ mass is determined by both quark and gluon condensates. Rough estimates based on the QCD sum-rule method have produced the estimate [6,7]

$$\frac{m_\rho^*}{m_\rho}\sim\left[\frac{\langle \bar{q}q\rangle^*}{\langle \bar{q}q\rangle}\right]^{1/3}, \tag{1}$$

which would point to an interchangeability of the ρ-meson mass and quark condensate as order parameters. We would like to present here a more thorough study of this problem, using the method of finite-temperature QCD sum rules. We should find that (1) is in fact only approximately true. In general, the order parameters $\langle \bar{q}q\rangle^*$ and m_ρ^* can have different critical types of behavior despite being related by the QCD sum-rule formula, and thus violations of the scaling relation (1) would single out one or the other quantity. Remarkably, we will find that the scaling violations are small and that (1) is still approximately valid.

We will limit ourselves to finite temperature only and leave density effects to a further study. Finite-temperature sum rules were introduced in [8] and elaborated in [9,10]. We will not give here any details of the procedure, but rather use the results of [10] for the calculation of the finite-temperature Wilson coefficients and proceed from there. We will depart from [10] when calculating the ρ-meson mass by way of the so-called "ratio method," which is more convenient for our purposes as it eliminates the dependence on the ρ-meson coupling strength f_ρ. Using this method, the final expression for the ρ mass then depends only on temperature, the Borel parameter M, the continuum threshold s_0, and of course the condensates, which we will use as input. We will comment on the dependence on the continuum threshold below.

The QCD sum-rule method at finite temperature is based on the fixed $|\mathbf{q}|$ dispersion relation for the vector correlator of currents:

$$\Pi_{00}(q_0,|\mathbf{q}|)=\frac{1}{\pi}\int\frac{\mathrm{Im}\Pi_{00}(\omega,|\mathbf{q}|)}{\omega-q_0}d\omega\ . \tag{2}$$

Here we have written down the dispersion relation for the 00 component of the correlator. Because of the lack of Lorentz invariance as a result of the presence of a heat bath, there is a transverse as well as a longitudinal form factor, the latter being related to Π_{00}. In the limit $|\mathbf{q}|\to 0$, which will be taken throughout, the two form factors turn out to be proportional, however. As is standard, the left-hand side (LHS) of (2) is evaluated at large spacelike momentum transfer, $-q_0^2\equiv Q^2\to\infty$, whereas the right-hand side (RHS) is parametrized as narrow resonance plus continuum, reflecting the possibilities of damping a vector excitation in the medium. As is well known, at finite temperature, there is, in addition to the classic channels of resonance formation and pair production, the possibility of Landau damping at vanishing momentum transfer. These contributions to the imaginary part of the vector correlator are depicted diagram-

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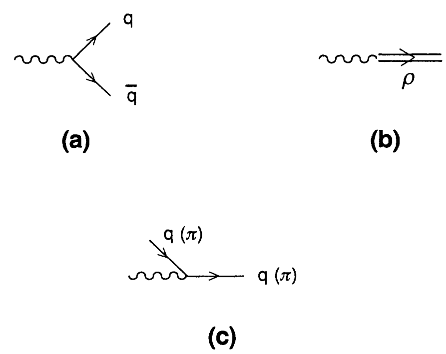

FIG. 1. Absorptive processes contributing to the imaginary part of the correlator to lead: (a) quark-antiquark pair formation, (b) ρ-resonance formation, and (c) Landau damping [virtual-quark (pion) absorption].

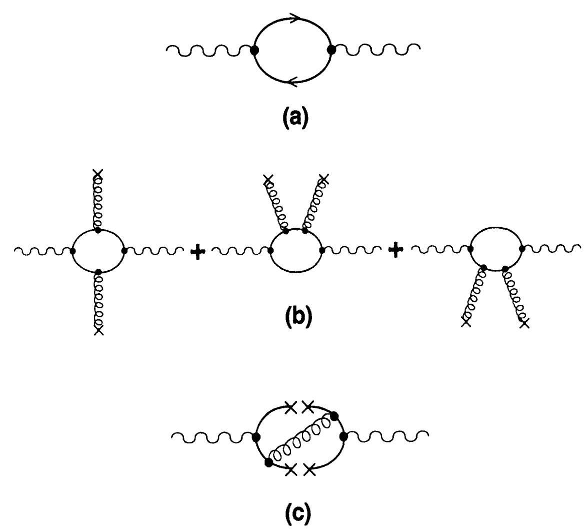

FIG. 2. (a) Leading perturbative contribution in the OPE, coefficient $\mathcal{C}_1$; (b) contribution of the gluon condensates $\mathcal{C}_{\mathbf{E}^2}$ and $\mathcal{C}_{\mathbf{B}^2}$; (c) contribution of the square of the quark condensate $\mathcal{C}_{(\bar{q}q)^2}$.

matically in Fig. 1. After the standard Borel transform we obtain, for the RHS of (2) (see [10]),[1]

$$\mathrm{RHS}=f_\rho \frac{m_\rho^2}{M^2} e^{-m_\rho^2/M^2} + \frac{1}{8\pi^2}\int_{s_0}^{\infty} \tanh\left[\frac{\omega}{4T}\right] e^{-\omega^2/M^2} d\omega^2 + c_L \frac{T^2}{M^2} . \quad (3)$$

On the other hand, a calculation of the LHS of (2) using the operator-product-expansion (OPE) technique in a nonperturbative temperature vacuum, keeping the diagrams in Fig. 2 [expansion to order $(M^2)^{-3}$], yields

$$\mathrm{LHS}=\mathcal{C}_1(T^2,M^2) + \mathcal{C}_{G^2}(T^2,M^2)\left[\frac{8}{3}\frac{\langle \mathbf{B}^2\rangle}{M^4} - \frac{4}{3}\frac{\langle \mathbf{E}^2\rangle}{M^4}\right] + \mathcal{C}_{(\bar{q}q)^2}\frac{\langle \bar{q}q\rangle^2}{M^6} , \quad (4)$$

where

$$\mathcal{C}_1(T^2,M^2)=\frac{1}{8\pi^2}\int_0^\infty \frac{d\omega^2}{M^2}\left[\tanh\left[\frac{\omega}{4T}\right] e^{-\omega^2/M^2} + 2n_F\left[\frac{\omega}{2T}\right]\right] , \quad (5)$$

$$\mathcal{C}_{G^2}(T^2,M^2)=\frac{\alpha_s}{24\pi}M^2\int_0^\infty \frac{d\omega^2}{\omega^4}\tanh\left[\frac{\omega}{4T}\right]\left[e^{-\omega^2/M^2}\left[1+\frac{\omega^2}{M^2}+2\frac{\omega^4}{M^4}\right]-1\right] , \quad (6)$$

$$\mathcal{C}_{(\bar{q}q)^2}=-\tfrac{56}{81}\pi\alpha_s , \quad (7)$$

and

$$n_F(\omega/T)=[1+\exp(\omega/T)]^{-1} .$$

Note that we have removed the tadpoles in (5) by doing one subtraction.

Equating (3) and (4), we obtain the sum rule

$$f_\rho \frac{m_\rho^2}{M^2} e^{-m_\rho^2/M^2} = R(M^2,T^2) , \quad (8)$$

[1]For the coefficient c_L in front of the last term, which is due to the Landau damping mechanism [see Fig. 1(c)], we obtain for a heat bath of quarks and gluons $c_L=-\frac{1}{2}$, while it is $+\frac{1}{3}$ for the case of damping through pion, rather than quark, absorption. We adopt the pionic Landau-damping scenario throughout. These coefficients differ from the estimates of [8,9,10], but are consistent with what is expected from the quark number susceptibility $\chi=-\Pi_{00}(0,\mathbf{0})$ on general grounds.

where we defined

$$R(T^2,M^2)=\mathcal{C}_1(T^2,M^2)-\frac{1}{8\pi^2}\int_{s_0}^{\infty}\tanh\left[\frac{\omega}{4T}\right]e^{-\omega^2/M^2}d\omega^2+c_L\frac{T^2}{M^2}+\mathcal{C}_{G^2}(T^2,M^2)\left[\frac{8}{3}\frac{\langle \mathbf{B}^2\rangle}{M^4}-\frac{4}{3}\frac{\langle \mathbf{E}^2\rangle}{M^4}\right]+\mathcal{C}_{\langle\bar{q}q\rangle^2}\frac{\langle\bar{q}q\rangle^2}{M^6}\ . \tag{9}$$

The coupling strength f_ρ is eliminated from (8) by taking the $1/M^2$ derivative and dividing by M^2R:

$$m_\rho^2(T^2,M^2)=-\frac{\partial}{\partial(1/M^2)}[M^2R(T^2,M^2)]/(M^2R)\ . \tag{10}$$

Specifically, we obtain

$$m_\rho^2=\frac{M^4I_1(M,T)-(\frac{8}{3}\langle\mathbf{B}^2\rangle-\frac{4}{3}\langle\mathbf{E}^2\rangle)I_2(M,T)+\frac{112}{81}\pi\alpha_s\langle\bar{q}q\rangle^2/M^2}{M^2I_3(M,T)+(\frac{1}{6}+c_L)T^2+I_4(M,T)(\frac{8}{3}\langle\mathbf{B}^2\rangle/M^2-\frac{4}{3}\langle\mathbf{E}^2\rangle/M^2)-\frac{56}{81}\pi\alpha_s\langle\bar{q}q\rangle^2/M^4}\ , \tag{11}$$

where the I_n are integrals of the order $\tanh(M/4T)$, which can easily be obtained from (5)–(10). Before investigating (11) numerically, we would like to make some qualitative comments. First, taken at face value, Eq. (11) suggests that a simple formula such as Eq. (1) cannot hold, as the gluon condensate contribution might turn out to be non-negligible at high temperatures, when the quark condensate contribution is negligible. Furthermore, while the quark condensate contribution adds to the mass ("repulsive contribution"), the gluon condensate subtracts from it ("attractive"). As we mentioned earlier, the gluon condensate itself, as opposed to the quark condensate, persists above T_χ. Therefore, as the quark condensate diminishes, those contributions could very well cancel, leading to a premature, first-order-like vanishing of the ρ mass before reaching the chiral-symmetry-restoring temperature, in contradiction to lattice-gauge-theory results [11], which indicate that the chiral-restoration transition involves a smooth crossover of thermodynamic variables. This contradiction is prevented by the coefficient of the gluon condensate contribution $\mathcal{C}_{G^2}$ going to zero as the Borel mass tends to zero. The latter can be shown to occur as a consequence of (11), as the minimum of the equation shifts to lower Borel masses when the quark condensate is reduced. This is consistent with the general idea [12] that the Borel mass parameter should roughly coincide with the resonance mass.

In the gluonic sector we are thus witnessing a situation which is opposite to the one in the quark sector: While $\mathcal{C}_{\langle\bar{q}q\rangle^2}$ is independent of temperature and the quark condensate drops with temperature, the converse is true for the gluonic Wilson coefficient and condensate. We hasten to add, though, that choosing a different vacuum (a different normal-ordering procedure for the operators) could well reverse the situation, as this may shift temperature dependences from condensates to Wilson coefficients and vice versa (see, e.g., [13]).

We have plotted the behavior of $\mathcal{C}_{G^2}$ as a function of M/T in Fig. 3 (this is essentially integral I_4 normalized to its zero-temperature value; its derivative, the integral I_2, has a very similar behavior) and the prediction of Eq. (11) in Fig. 4. To obtain the latter we have parametrized the decrease of the quark condensate assuming a second-order behavior of the form

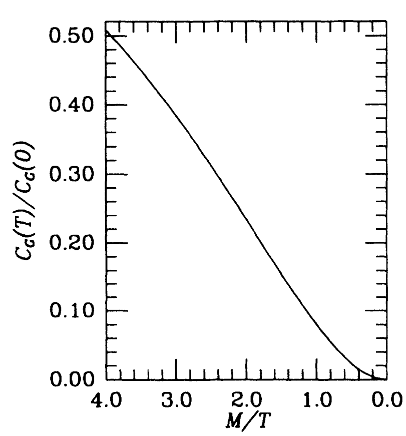

FIG. 3. Dependence of $\mathcal{C}_{G^2}$ [integral $I_4(M/T)$ in (11)] on M/T, normalized to its zero-temperature value.

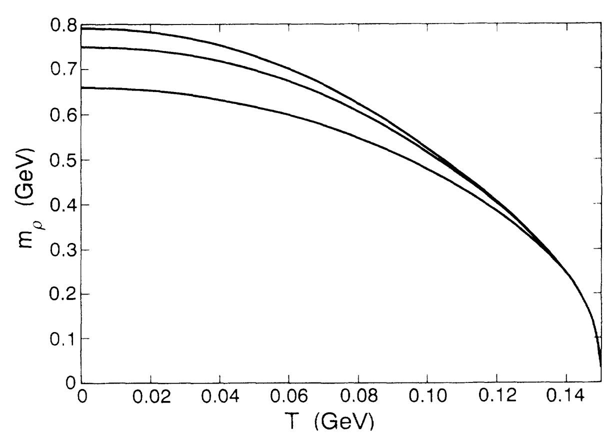

FIG. 4. ρ mass vs temperature as obtained from (11) for different continuum thresholds. The upper curve has $\sqrt{s_0}=1.5$ GeV, and the middle one is for $\sqrt{s_0}=1.25$ GeV, while the lower curve was calculated with $\sqrt{s_0}=1.0$ GeV as an input.

$$\frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} = [1-(T/T_\chi)^2]^{1/2} \tag{12}$$

and read off the ρ mass at the minimum of Eq. (11) at every temperature. This ensures that in this region the ρ mass is independent of the choice of Borel mass parameter. It is generally assumed (see also [8,10]) that the threshold drops with temperature. As we have no control over this issue when opting for the ratio method, we have performed the calculation for three different values for the continuum threshold s_0, taking $\sqrt{s_0}=1.5$, 1.25, and 1.0 GeV. Note that a higher continuum threshold implies a higher ρ mass only at small and moderate temperatures (see Fig. 4). The generally accepted zero-temperature value is $\sqrt{s_0}=1.5$ GeV (upper curve). The ρ mass becomes less and less dependent on the threshold as temperatures tend toward T_χ.

Of course, the precise profile of the finite-temperature quark condensate is not known; however, this is not needed to investigate the qualitative behavior of the ρ mass. Also, the value of higher-order condensates close to T_χ is not known, and those might change the picture close to the critical temperature. Another critical point is our use of a quark-gluon heat bath to calculate the temperature dependence of the Wilson coefficients [LHS of Eq. (2)]. While this approach is of questionable merit at very low temperatures, where the relevant degrees of freedom are expected to be hadronic, it turns out that the effect is also very small at these temperatures. The approach seems to be more reliable at temperatures close to T_χ, which is our region of interest. A calculation of ρ-meson parameters in the QCD sum-rule approach using a pionic heat bath to calculate temperature-dependent Wilson coefficients was performed very recently [14], yielding similar results at low and intermediate temperatures.

To sum up, the commensurate decrease of quark and gluonic contributions leads to a smooth behavior of the ρ mass up to T_χ. As this can be well described by Eq. (1), this seems to imply that the physics close to T_χ is, after all, controlled by just one scale.

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