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Isospin breaking in nuclear physics: the Nolen-Schiffer effect*

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Using the QCD sum rules we calculate the neutron-proton mass difference at zero density as a function of the difference in bare quark mass $m_d - m_u$. We confirm results of Hatsuda, Høgaasen and Prakash that the largest term results from the difference in up and down quark condensates, the explicit $\mathscr{C}(m_d - m_u)$ entering with the opposite sign. The quark condensates are then extended to finite density to estimate the Nolen-Schiffer effect. The neutron-proton mass difference is extremely density dependent, going to zero at roughly nuclear matter density.

The Ioffe formula for the nucleon mass is interpreted as a derivation, within the QCD sum rule approach, of the Nambu-Jona-Lasinio formula. This clarifies the N_c counting and furthermore provides an alternative interpretation of the Borel mass.

We compare calculations in the constituent quark model treated in the Nambu-Jona-Lasinio formalism with ours in the QCD sum rule approach.

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1. Introduction

The Nolen-Schiffer effect [1] has defied explanation for two decades. Recently, Henley and Krein [2] suggested a possible explanation in terms of dynamical mass generation, using the Nambu-Jona-Lasinio model. Hatsuda, Høgaasen and Prakash [3] realized that the calculations in the QCD sum rule formalism for the $\Xi - \Sigma$ energy splitting, which were carried out to linear order in the strange quark mass, could be immediately applied to the neutron-proton mass difference m_{np} by changing the strange quark mass m_s to m_d, the down quark mass. These authors found that

$$m_{np} = -4.79\,\gamma\,|\langle \bar{q}q\rangle|^{1/3} - 1.56\,(m_d - m_u), \tag{1.1}$$

where

$$\gamma = \frac{\langle \bar{d}d\rangle}{\langle \bar{u}u\rangle} - 1 \tag{1.2}$$

is negative, since the value of the quark condensate decreases with increasing current quark mass. Using the Ioffe formula [4] for the nucleon mass,

$$m_n \cong [-2(2\pi)^2 \langle \bar{q}q\rangle]^{1/3} \tag{1.3}$$

and the assumption that $\langle \bar{q}q\rangle$ scales with density like the nucleon effective mass

$$\frac{\langle \bar{q}q\rangle_\rho}{\langle \bar{q}q\rangle_0} = \left(\frac{m_n^*}{m_n}\right)^3 \tag{1.4}$$

these authors could quantitatively explain the Nolen-Schiffer effect. Especially noteworthy of (1.1) is that the condensate and bare quark mass difference enter with opposite signs, showing that the condensate plays a large role in m_{np}. With the Hatsuda et al. [3] $\gamma = -0.0065$, the neutron-proton mass difference (exclusive of electromagnetic effects) is 1.8 MeV for $\rho = 0$, given $m_d - m_u = 4$ MeV. This means that (putting the numbers in Hatsuda et al.'s eq. (5.9))

$$m_{np} \cong \left[8\,\frac{m_n^*}{m_n} - 6.2\right]\ \text{MeV} \tag{1.5}$$

so that $m_{np} \sim 0$ at nuclear matter density where $m_n^*/m_n \sim 0.8$. Thus, the neutron-proton mass difference changes sign in going to higher density. Henley and Krein [2] obtained a similar result.

The QCD sum rules provide a consistent formalism for the necessary calculations. We wish to show that, within this formalism, the different signs of dynamically generated mass and current quark mass effects emerge in a simple way. Whereas we agree with the results of Hatsuda et al. [3] we differ with the results of Belyaev and Ioffe [5] for the analogous $\Xi - \Sigma$ mass splitting. QCD

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sum rules at finite density have also been used previously to obtain the properties of nuclear matter [14].

The QCD sum rules focus on the high momentum, or short range behaviour, using a Borel transformation to extend calculations in the perturbative sector down to the nonperturbative one. We shall see that the main part of our results are determined by how one can make loops out of the quarks in the correlator of nucleon currents, these giving the leading $\ln\left(-\frac{q^2}{\mu^2}\right)$ behavior. Although the calculations are straightforward, the bare quark mass enters in a way which is counter intuitive from a low energy point of view. We shall try to explain, from a simple approach, how each of the isospin breaking terms enter.

Since the numerical results of Hatsuda et al. [3] vary with variations of the Borel mass M, one might feel somewhat uncomfortable with them. By our simple model we obtain quite similar results, that do not depend on the value of the Borel mass. This gives us confidence that they really have an explanation for the Nolen-Schiffer effect.

2. Isospin breaking in the QCD sum rules

We begin by outlining the simplest treatment of the nucleon giving the so called Ioffe formula [4], which retains only the quark condensates. The contribution of the gluon condensate is relatively small, and, in any case these will not contribute to the isospin breaking which we will discuss later in this section by introducing a difference $m_d - m_u$ between down and up quark masses.

The correlator of nucleon currents

$$\Pi(q^2) = i \int d^4x e^{iq\cdot x} \langle T\eta_N(x)\bar{\eta}_N(0)\rangle \tag{2.1}$$

can be decomposed into two invariant structure functions

$$\Pi(q^2) = \not{q}\Pi_1(q^2) + \Pi_2(q^2) \tag{2.2}$$

In Fig. 1 we show the contributions to be kept in the zero-order treatment. With a suitably chosen (proton) current[1] (C is the charge conjugation operator)

$$\eta_p(x) = \varepsilon^{abc}[\bar{u}(x) C\gamma_\mu u(x)][\gamma_5\gamma_\mu d(x)] \tag{2.3}$$

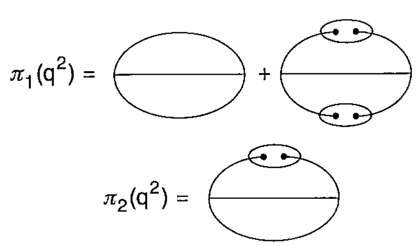

Fig. 1. Perturbative contribution and condensates included in the Ioffe formula

[1] This current, while not unique, has all the required properties of a nucleon current [4]. We shall assume in the following that it has enough overlap with the physical nucleon current that the results do not depend on the details of the source

it is easy to show that

$$\Pi_1(q^2) = -\frac{(-q^2)^2}{64\pi^4}\ln\left(-\frac{q^2}{\mu^2}\right) - \tfrac{2}{3}\langle\bar{q}q\rangle^2\frac{1}{q^2}$$
$$= -\frac{Q^4}{64\pi^4}\ln\left(\frac{Q^2}{\mu^2}\right) + \tfrac{2}{3}\langle\bar{q}q\rangle^2\frac{1}{Q^2} \tag{2.4}$$

$$\Pi_2(q^2) = -\frac{1}{4\pi^2}\langle\bar{q}q\rangle(-q^2)\ln\left(-\frac{q^2}{\mu^2}\right)$$
$$= -\frac{Q^2}{4\pi^2}\langle\bar{q}q\rangle\ln\left(\frac{Q^2}{\mu^2}\right) \tag{2.5}$$

where $Q^2 = -q^2$. For each Lorentz structure we have the sum rule

$$\Pi_{1,2}(Q^2) = \frac{1}{\pi}\int_0^\infty \frac{\operatorname{Im}\Pi_{1,2}(s)}{s+Q^2}\,ds \tag{2.6}$$

where we have left out possible subtractions, which will be taken care of by the Borel transform. After the Borel transform, the sum rules for Π_1 and Π_2 are:

$$\hat{L}_M\Pi_1(Q^2) = \frac{M^4}{32\pi^4} + \tfrac{2}{3}\frac{\langle\bar{q}q\rangle^2}{M^2}$$
$$= \lambda_N^2\frac{1}{M^2}e^{-m_n^2/M^2} \tag{2.7}$$

$$\hat{L}_M\Pi_2(Q^2) = -\frac{M^2}{4\pi^2}\langle\bar{q}q\rangle$$
$$= \lambda_N^2\frac{m_n}{M^2}e^{-m_n^2/M^2} \tag{2.8}$$

where $\hat{L}_M$ denotes the operator of the Borel transform

$$\hat{L} = \lim_{\substack{Q^2, n\to\infty \\ Q^2/n = M^2}} \frac{1}{(n-1)!}[Q^2]^n\left[-\frac{\partial}{\partial Q^2}\right]^n \tag{2.9}$$

where M^2 (the square of the Borel mass) is kept fixed, and λ_N is the overlap between correlator and nucleon state. Taking the ratio of (2.8) to (2.7) one has

$$m_n = \frac{2aM^4}{M^6 + \frac{4}{3}a^2} \tag{2.10}$$

where

$$a = -(2\pi)^2\langle\bar{q}q\rangle \tag{2.11}$$

The Ioffe formula (1.3) is obtained by neglecting the term $(4/3)a^2$ in the denominator of (2.10) and choosing $M = m_n$. In fact, the formula is much better than our rough description would suggest, because a continuum contribution which we have left out comes in to cancel a substantial part of the $(4/3)a^2$ in the denominator.

The above simple treatment gives us sufficient basis for studying the effect of quark condensates. As we show in Appendix A, the down quark condensate is somewhat smaller than the up quark one because of the larger down quark mass. For $m_d - m_u = 4$ MeV, we have

$$\gamma = \frac{\langle \bar{d}d \rangle}{\langle \bar{u}u \rangle} - 1 \cong -0.0065 \tag{2.12}$$

according to [3] (see also our estimate in Appendix A).

We now write down the full $\hat{L}_M \Pi(Q^2)$ for the proton, keeping the bare quark masses m_d and m_u to linear order

$$\hat{L}_M \Pi_1(Q^2) = \frac{M^4}{32\pi^4} + \frac{2}{3}\frac{\langle \bar{u}u \rangle^2}{M^2} + m_d \frac{\langle \bar{d}d \rangle}{4\pi^2} \tag{2.13}$$

$$\hat{L}_M \Pi_2(Q^2) = -\frac{M^2}{4\pi^2}\langle \bar{d}d \rangle - \frac{m_d}{16\pi^4} M^4 + \frac{4}{3}\frac{\langle \bar{u}u \rangle^2}{M^2}\left(m_d + \frac{2}{3} m_u (1+\gamma)\right) \tag{2.14}$$

In deriving these we have included the diagrams of Fig. 2. The results for each diagram alone are listed in

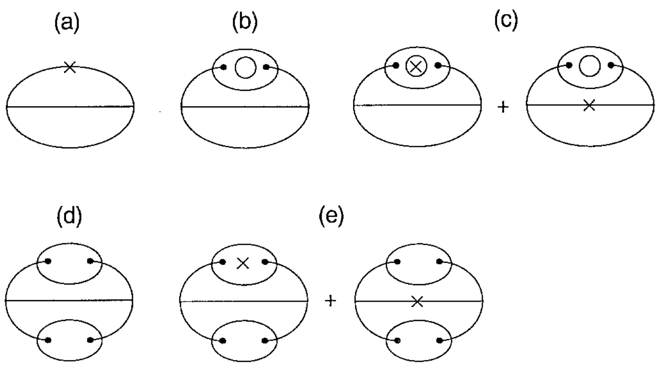

Fig. 2. Nonvanishing diagrams for the proton (Σ). The x denotes a heavy quark mass insertion; o denotes a heavy condensate. Diagrams a–e refers to the contributions listed in Table 1

Table 1. Contributions to the correlator from condensates and quark masses. The superscripts refer to the figures. $\langle \bar{q}q \rangle$ denotes the light quark condensate, the heavy quark condensate is $\langle \bar{q}q \rangle (1+\gamma)$

$$\Pi^{(2a)} = \frac{m}{32\pi^4}(-q^2)^2 \ln\left(-\frac{q^2}{\mu^2}\right)$$

$$\Pi^{(2b)} = \frac{\langle \bar{q}q \rangle}{4\pi^2}(1+\gamma) q^2 \ln\left(-\frac{q^2}{\mu^2}\right)$$

$$\Pi^{(2c)} = -\not{q} m \frac{\langle \bar{q}q \rangle (1+\gamma)}{4\pi^2} \ln\left(-\frac{q^2}{\mu^2}\right)$$

$$\Pi^{(2d)} = -\frac{2}{3}\not{q}\frac{\langle \bar{q}q \rangle^2}{q^2}$$

$$\Pi^{(2e)} = -\frac{4}{3} m \frac{\langle \bar{q}q \rangle^2}{q^2}$$

$$\Pi^{(3a)} = -\frac{2}{3}\not{q}\frac{\langle \bar{q}q \rangle^2}{q^2}(1+\gamma)^2$$

$$\Pi^{(3b)} = \frac{\langle \bar{q}q \rangle}{4\pi^2} q^2 \ln\left(-\frac{q^2}{\mu^2}\right)$$

$$\Pi^{(3c)} = -2m\frac{\langle \bar{q}q \rangle^2}{q^2}(1+\gamma)$$

Table 1. Of course, the $\langle \bar{d}d \rangle$ multiplying m_d can be taken to be the common $\langle \bar{q}q \rangle$ and the $\langle \bar{u}u \rangle^2$ can be taken to be $\langle \bar{q}q \rangle^2$ to the necessary order. We shall spend the rest of the chapter discussing the origin of the various terms.

(i) Most important for the condensate is the fact that the term $-(M^2/4\pi^2)\langle \bar{d}d \rangle$ for the proton, the term that gave the Ioffe formula, involves the down quark condensate only. This will give a contribution to the neutron-proton mass difference

$$\delta m_n^{(i)} \cong -2(2\pi^2)^2[\langle \bar{u}u \rangle - \langle \bar{d}d \rangle]/M^2$$
$$\cong -\gamma[-2(2\pi)^2\langle \bar{u}u \rangle]^{1/3} \cong -\gamma m_n \tag{2.15}$$

The reason that this term in Π_2 for the proton involves only $\langle \bar{d}d \rangle$ can be seen from Fig. 1. The condensate must be in the down quarks if there is to be a non flavour-changing loop (which gives rise to the $\log(Q^2/\mu^2)$). The reason why no flavour-changing loops appear (to linear order in the quark mass) is apparent from the structure of the nucleon current, (2.3). Since the current is essentially proportional to the d-quark field times a trace over the two same quarks, a condensate in one of the u-quark lines implies a trace over a single quark propagator, which vanishes. The cancellation of the higher order term $m_u \langle \bar{u}u \rangle$ (which appears in Π_1) is however nontrivial and might not be related to the symmetries of the current, as it only occurs in $D=4$. We have checked, however, that it also occurs for other choices of the nucleon source.

Note that with our estimate of $-\gamma \cong 0.0066$ in Appendix A, $\delta m_n^{(i)} \sim 7$ MeV, much greater than the empirical neutron-proton mass difference.

This result is much larger than we might intuit from a constituent quark model, where we would use the constituent quark mass $m_Q \sim \frac{1}{3} m_n$ instead of m_n in (2.15), the neutron and proton differing by one down quark. (We might also easily obtain the opposite sign for the above effect in the constituent quark model.)

(ii) From (2.14) it is clear that the term $\frac{4}{3}a^2$ in the denominator of (2.10) is to be taken as

$$\frac{4}{3}a^2 = \frac{4}{3}(2\pi)^4 \langle \bar{u}u \rangle^2 \tag{2.16}$$

in the case of the proton, but $\frac{4}{3}(2\pi)^4\langle \bar{d}d \rangle^2$ in the case of the neutron. Remembering that the Ioffe formula for m_n is obtained from (2.13, 2.14) by setting $M = m_n$, we see that if we kept the $(4/3)a^2$, then we would have

$$m_n \cong \frac{2a}{M^2}\left(1 - \frac{4}{3}\frac{a^2}{M^6}\right). \tag{2.17}$$

We argued that much of the $\frac{4}{3}\frac{a^2}{M^6}$ is cancelled by continuum terms. We will here make the assumption that the continuum is isospin symmetric, in order to compare with other works [3, 5] which implicitly make the same assumption (we shall investigate different continuum thresholds for neutron and proton in another section). Then, since the $\langle \bar{d}d \rangle^2$ term in the proton *subtracts* form the mass, there will be a further contribution

$$\delta^{(ii)} m_n \cong -\frac{2}{3}\gamma m_n \tag{2.18}$$

as can be seen by replacing the Borel mass by the nucleon mass and using the Ioffe formula everywhere.

(iii) The $m_d\langle\bar{d}d\rangle$ term in the proton again involves only the down quark condensate so that the two up quarks can make up a loop, giving the $\log Q^2$ term. This is a dimension-four term and enters into $\Pi_1(Q^2)$ precisely as a gluon condensate $\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle$ (had we kept it) would, but with the opposite sign. Multiplying numerator and denominator by $-(2\pi)^3$, we have

$$\frac{m_d\langle\bar{d}d\rangle}{4\pi^2}=\frac{m_d}{-2(2\pi)^4}[-2(2\pi)^2\langle\bar{d}d\rangle] \cong \frac{m_d m_n}{-2(2\pi)^4} \tag{2.19}$$

to compare with the main term $M^4/(32\pi^4)$ in $\hat{L}_M\Pi_1(Q^2)$. Thus, in (2.10) the M^6 in the denominator is replaced by

$$M^6 \rightarrow M^6 - m_n m_d M^2 \tag{2.20}$$

so that, with $m_n \sim M$, it is clear from expanding the denominator in (2.10) to first order that this term adds $\sim m_d$ to the proton mass. Thus

$$\delta^{(iii)}m_{np} \cong -m_d. \tag{2.21}$$

(iv) Furthermore, we deal with the term $(m_d/64\pi^4)M^4$ in $\hat{L}_M\Pi_2(Q^2)$. In some ways this is the simplest term in the isospin breaking. It comes from the mass term in the bare quark propagator, that we can write in coordinate space as (to first order in the quark mass)

$$S_d(x)=-\frac{1}{2\pi^2}\left\{\frac{\gamma\cdot x}{x^4}+\frac{i}{2}\frac{m_d}{x^2}\right\}. \tag{2.22}$$

In the case of the proton only m_d enters if two of the quarks are to be contracted into a loop; since they must be identical to lowest order these must be the two up quarks. Multiplying and dividing the $(-M^2/4\pi^2)\langle\bar{d}d\rangle$ term by $2(2\pi)^2$ and using the Ioffe formula, we have

$$\hat{L}_m\Pi_2(Q^2)=\frac{M^2 m_n}{32\pi^4}+\frac{m_d M^4}{16\pi^4} +\frac{4}{3}\frac{\langle\bar{q}q\rangle^2}{M^2}\left(m_d+\frac{3}{2}m_u\right). \tag{2.23}$$

Thus, the $2a$ in the numerator of (2.10) is replaced by $2a+2m_d\left(M^2+\frac{4}{3}\frac{a^2}{M^4}\right)$ so that the mass of the proton is increased by $2m_d$. Consequently

$$\delta^{(iv)}m_{np} \cong -2m_d. \tag{2.24}$$

It is amusing that the m_d in the quark propagator increases the proton mass relative to the neutron, even though the latter has two down quarks, the former only one. As we have pointed out, the result is determined by how one has to make loops, which have the dominant (logarithmic) behavior in Q^2.

Finally, due to the different coefficient in front of the $m_q\langle\bar{q}q\rangle^2$ contribution for up and down quarks (see (2.14)), for the neutron the factor $2a$ in the numerator of (2.10) is replaced by $2a+2m_d\left(M^2+\frac{4}{3}\frac{a^2}{M^4}\cdot\frac{3}{2}\right)$, which results in an additional mass difference

$$\delta^{(v)}m_{np} \cong +m_d. \tag{2.25}$$

Of course, all five of the δm_{np} we have calculated thus far are linear in m_d, and only the sum of them is meaningful.

Our results are obtained by summing the δm_{np}:

$$m_{np} \cong -\frac{5}{3}\gamma m_n - 2(m_d - m_u) \tag{2.26}$$

where we have reinstated the difference in masses. With $\gamma = -0.0065$ this gives

$$m_{np}=\left[10.8\frac{m_n^*}{m_n}-8\right]\text{ MeV} \tag{2.27}$$

to compare with the formula of Hatsuda et al. (1.5). Although the coefficients we obtained using this rough and schematic way are somewhat larger than those obtained by the more detailed treatment of [3], we also obtain a nearly vanishing m_{np} at nuclear matter density, where $m^*/m \sim 0.8$, using $m_d - m_u = 4$ MeV. Note that when calculating m^*, the nucleon mass at finite density, we have kept only the contribution of the scalar condensate. It is well known however that finite density creates a non-zero vector condensate, which gives rise to a vector field, which shifts the physical nucleon mass back near to its zero density value. (It is known that the nucleon chemical potential μ is close to the rest mass m_n; therefore, as far as the nucleon energy is concerned, the vector and scalar fields largely cancel.) In the mass-*difference* however, the vector condensate appears only as a higher order correction. Since there is no vector condensate in the $\mu = 0$ vacuum, the shift from the vector potential can be accounted for in a straightforward manner. The impact of the vector condensate is discussed in more detail in [3].

Our first term does, however, agree with what we would obtain from the difference of Ξ and Σ masses, (4.72c) and (4.72d) of Reinders, Rubinstein and Yazaki [6], after taking the Borel mass M to be equal to m_n. Of course, both terms in either formula (1.5) or (2.27) depend linearly on $m_d - m_u$, so that after introduction of the electromagnetic mass differences, we can adjust $m_d - m_u$ slightly to obtain the correct neutron-proton mass difference at zero density.

We have confirmed the main point of Hatsuda et al. that the dynamically generated contribution and bare quark piece enter with opposite sign. This means that the coefficient of the dynamically generated contribution can be quite large, as it is in both Hatsuda et al. [3] and in our formula (2.27). In either case, the neutron proton mass difference will be substantially lessened in nuclei, changing sign for $\rho \sim \rho_0$, where ρ_0 is nuclear matter density.

3. Relation to the Nambu-Jona-Lasinio formula

In this section we wish to show the relation of the dynamical mass generation by the QCD sum rules to the Nambu-Jona-Lasinio model. We begin by keeping only the lowest terms in (2.10) resulting in the Ioffe formula

$$m_n = \frac{2a}{M^2} = \frac{-2(2\pi)^2 \langle \bar{q}q \rangle}{M^2}; \tag{3.1}$$

i.e. this equation would result in the Ioffe formula if the Borel mass M is replaced by m_n. We wish to offer another possibility.

The Nambu-Jona-Lasinio formula for m_n is usually written

$$m_n = -G\langle \bar{q}q \rangle \tag{3.2}$$

where the quark condensate is[2]

$$\langle \bar{q}q \rangle = -g \int_0^{\Lambda} \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{m_Q}{\sqrt{k^2+m_Q^2}} \tag{3.3}$$

with m_Q the constituent quark mass, g the degeneracy, and G a constant. Although Λ is considered a somewhat arbitrary cut off in the NJL formalism, since the magnitude of $\langle \bar{q}q \rangle$ is usually well known from the Gell-Mann-Oakes-Renner relation, we can determine[3] Λ in order to reproduce this known value of

$$\langle \bar{q}q \rangle \cong -(220\,\mathrm{MeV})^3. \tag{3.4}$$

It is convenient [7] to consider m_n to result from the mean field relation

$$m_n = -\tfrac{2}{3}\frac{g_{\sigma NN}^2}{m_\sigma^2}\langle \bar{q}q \rangle, \tag{3.5}$$

which is shown graphically in Fig. 4. In other words, we interpret G of (3.2) as

$$G = \tfrac{2}{3}\frac{g_{\sigma NN}^2}{m_\sigma^2}. \tag{3.6}$$

Of course, exchange terms (of relative order g^{-1} compared with the leading term), short range correlations, etc., can be included. They would change the value of m_σ.

In mean field approximation and tree level, m_σ was found to be $940\,\mathrm{MeV} \approx m_n$ in the estimate of [7]. We therefore suggest that the M^2 in the denominator of (3.2) should be interpreted as m_σ^2. Tracing back where M^2 came from, we see that the lowest order term, the perturbative quark loop in $\Pi_1(q^2)$ (see Fig. 1) goes as $Q^4 \ln Q^2$, whereas $\Pi_2(q^2)$ goes as $\langle \bar{q}q \rangle Q^2 \ln Q^2$. The $Q^4 \ln Q^2$ turns into M^4 and the $Q^2 \ln Q^2$ into M^2 upon Borel transforming. The Ioffe expression for m_n comes from the ratio of Π_2 to Π_1, so that the M^2 in the denominator of (3.2) arises from the diquark loop that $\Pi_1(q^2)$ has in addition to that of $\Pi_2(q^2)$. (The former has two loops, the latter one). This diquark loop is a scalar, not unlike the $\bar{q}q$ loop which would represent a scalar particle as $Q^2 \to \infty$, stripping off all gluon interactions.

Our interpretation simplifies the problem of N_c dependence. Both m_n and $\langle \bar{q}q \rangle$ are linear in N_c, whereas m_σ has no N_c dependence. Thus, both sides of (3.2) have the same (linear) N_c dependence. This is not apparent in the Ioffe formula, (1.3).

If we identify m_σ with M, then from (3.5) we find, comparing with (3.1)

$$\frac{g_{\sigma NN}^2}{4\pi} = 3\pi. \tag{3.7}$$

This is obtained in tree approximation, so that to obtain the pion-nucleon coupling constant we must multiply by g_A^2, which gives

$$\frac{g_{\pi NN}^2}{4\pi} = 3\pi g_A^2 = 14.7 \tag{3.8}$$

where we have used $g_A = 1.25$. At the very least, this gives an amusing interpretation of the factor $2(2\pi)^2$ in the Ioffe formula, (3.1).

Let us continue our considerations to finite density. Taking all masses, except the pion mass, to scale [7] as f_π^*; i.e.

$$\frac{m_n^*}{m_n} = \frac{m_\sigma^*}{m_\sigma} = \frac{f_\pi^*}{f_\pi} \quad \text{and} \quad \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} = \frac{(f_\pi^*)^3}{f_\pi^3} \tag{3.9}$$

the NJL formula goes into

$$m_n^* = \frac{-\frac{2}{3}g_{\sigma NN}^2}{(m_\sigma^*)^2}\langle \bar{q}q \rangle^*. \tag{3.10}$$

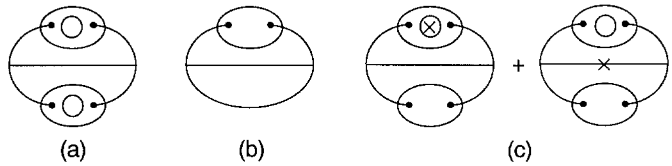

Fig. 3. Same as Fig. 2, but for the neutron (Ξ)

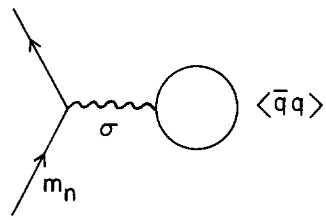

Fig .4. Graphical representation of the mean field relation considered behind NJL

[2] In this section we are neglecting the bare quark mass as it is not essential for these arguments. The full expression including effects from the bare quark mass can be found in Appendix A

[3] In order to compare results with [2] we shall frequently use $\Lambda = 0.8$ MeV, which does *not* reproduce (3.4)

By $\langle \bar{q}q \rangle^*$ we mean that on the right hand side of (3.3) not only $m_Q \to m_Q^*$, but also $\Lambda \to \Lambda^*$, where

$$\frac{\Lambda^*}{\Lambda} = \frac{f_\pi^*}{f_\pi}. \tag{3.11}$$

However, we can rewrite (3.10) as

$$m_n^* = -\frac{2}{3} \frac{g_{\sigma NN}^2}{m_\sigma^2} \frac{\langle \bar{q}q \rangle^*}{(f_\pi^*/f_\pi)^2}$$

$$= -G \frac{\langle \bar{q}q \rangle^*}{(f_\pi^*/f_\pi)^2} \tag{3.12}$$

using (3.9).

In practice $\Lambda \sim 800$ MeV, $m_Q \sim 330$ MeV so that $\Lambda \gg m_Q$. Thus, to a good approximation

$$\langle \bar{q}q \rangle \cong -\mathscr{C} \Lambda^2 m_Q \tag{3.13}$$

in dependence on Λ and m_Q, and similarly $\langle \bar{q}q \rangle^* \cong -\mathscr{C} \Lambda^{*2} m_Q^*$. Using (3.11) we find

$$\frac{\langle \bar{q}q \rangle^*}{(f_\pi^*/f_\pi)^2} \cong -\mathscr{C} \frac{\Lambda^{*2} m_Q^*}{(f_\pi^*/f_\pi)^2} = -\mathscr{C} \Lambda^2 m_Q^*. \tag{3.14}$$

Defining

$$\langle \bar{q}q \rangle_{\mathrm{NJL}}^* = -g \int_0^\Lambda \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{m_Q^*}{\sqrt{k^2 + m_Q^{*2}}} \tag{3.15}$$

we see that

$$m_n^* = -G \langle \bar{q}q \rangle_{\mathrm{NJL}}^* \tag{3.16}$$

to within terms of relative order m_Q^2/Λ^2. Thus, to a good approximation, the Nambu-Jona-Lasinio formula (3.2) can be continued in medium by letting $m_n \to m_n^*$ and $m_Q \to m_Q^*$ and keeping Λ fixed.

Thus far no calculation of how $(g_{\sigma NN}^*)^2$ changes going into the medium exists. Calculations [7] of the medium dependence of $(g_{\rho NN}^*)^2$ show that it decreases at less than or about half the rate of $(m_\rho^*)^2$ as the density ρ goes from 0 to nuclear matter density ρ_0. We plan to address this issue in the future. In general calculations in the tree approximation in the literature [8, 9] show $g_{\sigma NN}^*$ to change little with density or temperature. This may be an artefact of not including loops. For example, g_A increases from 1 to 1.26 with loop corrections, whereas g_A^* decreases from 1.26 in free space to ~ 1 as measured [10] in (s, d)-shell nuclei. Arguments have been [7] made that $g_{\pi NN}^*$ should scale with density as g_A^* for not too high momenta $p \ll m_{A_1}$, where m_{A_1} is the A_1-meson mass. From chiral invariance, one might expect $g_{\sigma NN}^*$ to scale as $g_{\pi NN}^*$. Thus, loop corrections might introduce appreciable density dependence in $g_{\sigma NN}^*$.

On the other hand, our connection above of the Ioffe formula with Nambu-Jona-Lasinio was made at tree level, indicating that loop corrections must be found in higher order terms in the QCD sum rules. Whereas the connection between the scaling of g_A^* and $g_{\pi NN}^*$ in [7] is made from low momenta, and the Goldberger-Treiman relation holds for zero momentum, the QCD sum rules let the momentum go to infinity. Thus far, connections between the results from calculations in these two regimes have proved difficult to make.

4. Discussion of the Nolen-Schiffer effect

Hatsuda, Høgassen and Prakash [3] neglected the density dependence of γ, (1.2) in stating their final results, although this density dependence as calculated in the NJL formalism and shown in their Fig. 2 is fully as rapid as in m_n^*, i.e.

$$\frac{\gamma^*}{\gamma} \approx \frac{m_n^*}{m_n}. \tag{4.1}$$

Inclusion of this additional density dependence would nearly double their effect. Since we have now connected the NJL formalism with the Ioffe formula, we feel justified in using the former in the QCD sum rule framework.

Indeed, for $\Lambda = 800$ MeV and $m_Q = 330$ MeV, to order m_Q^2/Λ^2 it is easy to calculate the change in $\langle \bar{q}q \rangle_{\mathrm{NJL}}^*$ due to introduction of a small bare quark mass $m^{(0)}$ as (see Appendix A)

$$\frac{\delta \langle \bar{q}q \rangle_{\mathrm{NJL}}^*}{\delta m^{(0)}} \cong 0.49\, m_Q^{*2} \tag{4.2}$$

where we take $m_Q^* \sim 0.8\, m_Q$. Note that this change comes from the low momentum region of the integrand in (3.15) and should be insensitive to the cutoff Λ. Using the definition (1.2) of γ, (3.14) and (4.2) we find that

$$\gamma^* = (m_d - m_u) \frac{\delta \ln \langle \bar{q}q \rangle_{\mathrm{NJL}}}{\delta m^{(0)}}$$

$$= \frac{-0.49\,(m_d - m_u)\, m_Q^*}{\mathscr{C} \Lambda^2}. \tag{4.3}$$

This essentially linar dependence of γ^* on m_Q^* has the consequence that the m_n^*/m_n in (1.5) and (2.27) are replaced by $(m_n^*/m_n)^2$, thus nearly doubling the Nolen-Schiffer effect.

After years of futile efforts to obtain sufficiently large quantities to explain the Nolen-Schiffer effect, it seems refreshing to, for a change, obtain too large an effect. On the other hand, the QCD sum rules concentrate on the high Q^2, or short range aspects, and the longer range terms would be expected to give much less of an effect.

Consider, for example, what happens in the constituent quark model, treated in NJL [2]. Instead of m_n, take the up or down constituent quark masses, M_u or M_d, to be generated by $\langle \bar{q}q \rangle$. Take the σ to be made up as

$$\sigma = \frac{1}{\sqrt{2}} \,|(\bar{u}u + \bar{d}d)\rangle. \tag{4.4}$$

Then, from the direct terms shown in Fig. 4 the generated masses M_u and M_d will be equal. With introduction of exchange terms, the $\langle\bar{u}u\rangle$ contribution to M_u will be diminished by g^{-1}, where g is the degeneracy ($g=6$), and the $\langle\bar{d}d\rangle$ contribution to M_d will be decreased. Thus, to order g^{-1}, the up quark mass comes slightly more from $\langle\bar{d}d\rangle$ than from $\langle\bar{u}u\rangle$, just as in the Ioffe formula the proton mass comes from $\langle\bar{d}d\rangle$ even though the proton contains two up quarks and only one down quark. However, in the neutron-proton mass difference m_{np}, there is a factor of $1/18$ in the constituent quark model compared with the Ioffe formula, $1/6$ of this being g^{-1}, and the other $1/3$ from the ratio of quark mass to nucleon mass. This discussion may help to explain why the QCD sum rule calculation gives such a large answer with the parameters used. As noted, to the extent that we are allowed to decrease the bare quark mass difference m_d-m_u, we can improve the quantitative argreement with the empirical effect.

5. Conclusions

We have confirmed the result of Hatsuda, Høgaasen and Prakash that the dynamically generated and bare quark contributions to m_{np} have opposite sign, leading to a large change in m_{np} with density. We have shown that this results from the way in which quark loops are contracted, in order to obtain the high Q^2, or short distance, behaviour.

Many partial explanations of the Nolen-Schiffer effect have been given within the language of the broken symmetry sector; i.e., mesons and nucleons. These have been in terms of electromagnetic effects, $\rho-\omega$-mixing, etc. [11, 12]. It may well be that by adding these effects up, one has a real explanation. This in no way contradicts our explanation, which uses different variables, especially the quark condensates. We suggest that effects like the Nolen-Schiffer one have explanations in both sectors. The reason that the QCD sum rules work is that the perturbative and the nonperturbative sectors overlap; thus, perhaps, a given phenomenon can be described by the variables in either sector. One example of this overlapping in explanations is the decrease in meson masses with density. In Brown, Müther and Prakash [13] the decreasing meson masses were described in terms of medium corrections which involve isobar nucleon-hole insertions in pion propagators. An explanation of the same phenomenon is given in [7] in terms of changes in the quark condensate with density. Although it is clear that both explanations rely on partial restoration of chiral symmetry with increasing density, they have not yet been as tightly connected as they should be.

We have reinterpreted the Ioffe formula to be a derivation, in the no loop approximation, of the NJL formula for the mass of the nucleon. The Borel mass finds a physical meaning here as the scalar meson mass m_σ and cannot be arbitrarily varied. Thus, we do not see the need for m_n to be insensitive (in the region where $M^2=m_n^2$) to variations in the Borel mass. The mass of the ρ-meson, m_ρ, is quite flat as a function of M in the region $M\sim m_\rho$ so that the ρ-meson is well accommodated in the philosophy of the QCD sum rules. It may be that the mechanism of mass generation for the ρ-meson is quite different from that for the nucleon, although most of the mass, in both cases, comes from the $\langle\bar{q}q\rangle$ condensate.

We pointed out that, in detail, the isospin breaking is much larger in the QCD sum rules than in the constituent quark picture, described in the Nambu-Jona-Lasinio formalism.

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Appendix

In this appendix we use the Nambu-Jona-Lasinio theory to determine γ. In the NJL theory the condensate $\langle\bar{q}q\rangle$ depends quadratically on the cut off. We shall see that γ depends only logarithmically on Λ, however, so we should be able to estimate γ reliably in the NJL formalism.

The condensate is just the scalar density in the negative energy sea, but with a subtraction such that the condensate vanishes if the dynamically generated mass is zero [8].

$$\begin{aligned}\langle\bar{q}q\rangle &= -g\int_0^\Lambda \frac{d^3k}{(2\pi)^3}\left\{\frac{(\tilde{m}+m^{(0)})}{\sqrt{k^2+(\tilde{m}+m^{(0)})^2}}-\frac{m^{(0)}}{\sqrt{k^2+(m^{(0)})^2}}\right\}\\ &= -\frac{(\tilde{m}+m^{(0)})}{4\pi^2}g\left[\Lambda\sqrt{\Lambda^2+(\tilde{m}+m^{(0)})^2}-(\tilde{m}+m^{(0)})^2\ln\frac{\Lambda+\sqrt{\Lambda^2+(\tilde{m}+m^{(0)})^2}}{\tilde{m}+m^{(0)}}\right]\\ &\quad+\frac{m^{(0)}g}{4\pi^2}\left[\Lambda\sqrt{\Lambda^2+(m^{(0)})^2}-(m^{(0)})^2\ln\frac{\Lambda+\sqrt{\Lambda^2+(m^{(0)})^2}}{m^{(0)}}\right]\end{aligned}\tag{A1}$$

Here, $g=6$ is the degeneracy. Also, $\tilde{m}=330$ MeV is the dynamically generated mass and $m^{(0)}$ is the bare quark mass. Thus, $m_Q=\tilde{m}+m^{(0)}$. Following Henley and Krein [2], we take $\Lambda=800$ MeV.

We first calculate $\langle\bar{q}q\rangle$ with $m^{(0)}=0$ (in the chiral limit). We find

$$\langle\bar{q}q\rangle=-(296\text{ MeV})^3.\tag{A2}$$

This is too high, due to the value of the cut off. However, as mentioned earlier, the ratio of condensates γ is much less sensitive to the cut off. Furthermore, we take $m_s^{(0)}=190$ MeV for the strange quark mass (with $m_Q=540$ MeV), which gives us

$$\langle\bar{s}s\rangle=-(263\text{ MeV})^3.\tag{A3}$$

Thus,

$$\frac{\langle\bar{s}s\rangle}{\langle\bar{u}u\rangle}\cong 0.71,\tag{A4}$$

in the generally assumed range. This means that

$$\gamma_s = \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} - 1 = -0.29. \tag{A5}$$

Using (2.26) we see that this implies for the $\Xi - \Sigma$ mass-difference

$$m_\Xi - m_\Sigma = -\tfrac{5}{3}(-0.71)\, m_n - 2 \cdot 190 \text{ MeV} = 81 \text{ MeV} \tag{A6}$$

to compare with the empirical 124 MeV.

Taking $m_d - m_u = 4$ MeV and $m_d = 7$ MeV we calculate γ of the neutron-proton system to be

$$\gamma = -0.0066. \tag{A7}$$

Note that scaling γ_s linearly with the bare quark mass would give

$$\gamma = \gamma_s \cdot \frac{4}{190} = -0.0062. \tag{A8}$$

Thus, at least for γ, linear perturbation theory works well. In fact, for small $m^{(0)}$, the linear theory gives

$$\frac{\delta \langle \bar{q}q \rangle}{\delta m^{(0)}} = 0.49\, (\tilde{m})^2. \tag{A9}$$

The nonanalytic term $\ln m^{(0)}$ in (A1) is multiplied by $(m^{(0)})^3$ so that, at least for small bare quark masses, there is no problem in practice with this expansion.

It is straightforward to see that $|\gamma|$ is a decreasing function of density. This dependence is shown in Fig. 2 of Hatsuda et al. [3] and in general our results in this Appendix are in agreement with these authors.

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