CHARMONIUM DISINTEGRATION BY FIELD IONIZATION

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Charmonium bound states immersed in a coherent chromo-electric field are easily ripped apart for field strengths comparable to the QCD string tension. Estimates based on flux tube models suggest that field strengths of such magnitude may be achieved in heavy-ion collisions at ultrarelativistic energies. Our results suggest that charmonium suppression would not discriminate between a coherent and a thermalized post collision state in relativistic heavy-ion collisions.

The possibility that hadronic matter might undergo a phase transition to a quark-gluon plasma (QGP) at high temperature and/or density, has attracted considerable interest both theoretically and experimentally [1]. Although not much is known about the dynamics of the phase transition, several distinct signatures of such a transition have been put forward. One that has drawn much attention recently is the possibility of a substantial suppression of the J/ψ resonance due to color screening in the QGP [2]. If the temperature dependent screening length becomes larger than the size of the bound state, the bound state ceases to exist. Although this mechanism accounts for J/ψ-suppression, it is not obvious that such a suppression is an unambiguous signature of QGP.

In the absence of clear experimental evidence for the phase transition, the assumption that a plasma is formed in a nucleus-nucleus collision depends crucially on assumptions about the mechanism of energy deposition and thermalization. For infinitely heavy quark sources, lattice calculations show a phase transition at about 200 MeV. The quarks in the J/ψ, however, are not that heavy for otherwise the system would be entirely coulombic and therefore insensitive to the confining mechanism. As yet, no conclusive predictions exist about the nature and character of the high-temperature phase of light bound states.

It is therefore imperative to investigate the environment in which the J/ψ is formed, independently of the assumption of plasma formation. Attempts in this direction have been made recently using hadronic interactions [3–5].

In this letter we wish to address another aspect of the collision environment, namely the effect of a coherently formed macroscopic color-electric field on the charmonium state. In a simple model of a nucleus-nucleus collision, every projectile nucleon interacts once with a target nucleon by the exchange of a single gluon. As the nucleons recede from each other, a gluonic flux tube is formed. If the one-gluon exchange processes occur coherently, these individual flux tubes can add up to a homogenous, macroscopic color-electric field [6–9]. This is of course an extreme point of view, where coherence is entertained in favor of a state with maximal entropy. Presumably the truth of the matter lies somewhere in between a thermalized state of maximum entropy (the QGP), and a state with minimum entropy (the flux tube) formed through coherent gluonic exchange. Ideally, one would like a clear signature for or against QGP formation to discriminate between the two extremes. Our purpose in this paper is to show that J/ψ suppression occurs also in a state of minimum entropy from a coherent exchange of gluons. Note that in such a scenario, thermalization or QGP formation are not required.

Consider a J/ψ in an environment characterized by a coherent colored flux tube. The bound state is

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described by a static, nonrelativistic confining hamiltonian

$$H = p^2 / \mu - \frac{3}{4} \alpha_s / r + \sigma r$$,

(1)

where $\alpha_s$ is the strong coupling constant, $\mu$ is the mass of the charm quark and $\sigma$ is the string tension. To estimate the effect of the external chromo-electric field on the $J/\psi$, we shall be guided by the abelian version of the interaction. Since this is the usual Stark effect, the interaction is

$$V_{\text{int}} = -eFz$$,

(2)

where $F = E_z$ is the field strength in the $z$-direction. For the color-dipole interaction we shall take $F \sim |E|$, where $E_z = G_{AB}^z$ is the color-electric field strength. We then normalize the field in such a way that

$$V_{\text{int}} = -\beta Fz$$,

(3)

where $\beta = \frac{3}{4} \alpha_s$. The Schrödinger equation for this system is then

$$(-\nabla^2 / \mu - \beta / r + \sigma - \beta F z) \Psi(r) = E \Psi(r)$$.

(4)

This equation is most easily investigated in parabolic coordinates. Introducing

$$\xi = r + z$$,  \hspace{1cm} (5)

$$\eta = r - z$$,  \hspace{1cm} (6)

$$\phi = \tan^{-1}(y/x)$$  \hspace{1cm} (7)

and

$$\Psi(r) = [u(\xi, \eta) / \sqrt{\xi \eta}] \exp(\pm i m \phi)$$,  \hspace{1cm} (8)

we find that the angular part separates, leaving us with the second order partial differential equation

$$\xi \partial^2 u / \partial \xi^2 + u / 4 \xi + \frac{1}{4} \mu \beta^{(1)} u - \frac{1}{2} \mu (\sigma - \beta F) \xi^2 u$$

$$+ \frac{1}{2} \mu E \xi u + \eta (\partial^2 u / \partial \eta^2 + u / 4 \eta + \frac{1}{4} \mu \beta^{(2)} - \frac{1}{2} \mu (\sigma + \beta F) \eta^2 u + \frac{1}{2} \mu E \eta u - \frac{1}{4} \mu \alpha \xi \eta u = 0$$.

(9)

Here, we have introduced $\beta = \frac{1}{2} (\beta^{(1)} + \beta^{(2)})$. In the following, we shall focus on the ground state, for which $m = 0$. The unperturbed state has a symmetric distribution of color charge, implying $\beta^{(1)} = \beta^{(2)} = \beta$. Turning on the color-electric field then introduces an asymmetry in the charge distribution. It turns out that neglecting the cross term in (9) does not affect the wavefunctions substantially, whence the equation separates further. Setting $u(\xi, \eta) = u_1(\xi) u_2(\eta)$ we find

$$u_1'' + u_1 / 4 \xi^2 + (\mu \beta^{(1)}) / 4 \xi^2 + \frac{1}{2} \beta (\sigma - \beta F) \xi u_1 + \frac{1}{2} \mu E u_1 = 0$$,

(10)

$$u_2'' + u_2 / 4 \eta^2 + (\mu \beta^{(2)}) / 4 \eta^2 + \frac{1}{2} \beta (\sigma + \beta F) \eta u_2$$

$$+ \frac{1}{2} \mu E u_2 = 0$$.

(11)

The parameters $\beta^{(1)}$ and $\beta^{(2)}$ now serve as eigenvalues of the problem. After solving (10) and (11) the energy is obtained via the requirement $\frac{1}{2} (\beta^{(1)} + \beta^{(2)}) = \beta$. From these equations it follows that $u_1(\xi)$ describes the part of the wavefunction which feels a weakened “string tension” due to the counterbalancing effect of the field, whereas $u_2(\eta)$ senses an effective string tension enhanced by the field. These wavefunctions describe dipole-like charge distributions that are oriented parallel (stretched) and antiparallel (squeezed) to the color-field, respectively. As long as the field is not too strong, the overall effect is a lowering of the energy of the ground state just as in the usual Stark effect on atomic levels.

If, however, $\beta F$ becomes comparable to the string tension, a region in space is created where the potential can take arbitrarily large negative values away from the core, providing a region to which one of the quarks can tunnel. This effect is analogous to the well-known field ionization of the hydrogen atom in very strong fields [10–12].

To see this, we rewrite eqs. (10) and (11) in the following convenient forms:

$$u_1'' + \Phi_-(\xi) u_1 = 0$$,  \hspace{1cm} (12)

$$u_2'' + \Phi_+(\eta) u_2 = 0$$,  \hspace{1cm} (13)

with obvious definitions. If we now introduce the dimensionless variables

$$x = (\mu \beta)^{-1} \xi$$,  \hspace{1cm} \eta = (\mu \beta)^{-1} \eta$$,  \hspace{1cm} \kappa = \mu^{-2} \beta^{-3} \sigma$$,

$$\vec{F} = (\mu \beta)^{-1} \vec{F}$$,  \hspace{1cm} \epsilon = \mu^{-1} \beta^{-2} \vec{E}$$,

(14)

eq (\mu \beta)^{-1} \xi$$,  \hspace{1cm} \eta = (\mu \beta)^{-1} \eta$$,  \hspace{1cm} \kappa = \mu^{-2} \beta^{-3} \sigma$$,

$$\vec{F} = (\mu \beta)^{-1} \vec{F}$$,  \hspace{1cm} \epsilon = \mu^{-1} \beta^{-2} \vec{E}$$,

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$$\vec{F} = (\mu \beta)^{-1} \vec{F}$$,  \hspace{1cm} \epsilon = \mu^{-1} \beta^{-2} \vec{E}$$,

(14)
\[ \Phi_+(y) = \frac{1}{4} [y^{-2} + (\beta^{(12)} / \beta) y^{-1} - \frac{1}{2} (\kappa + F) y + \epsilon] . \]

(18)

Above, \( \Phi_\pm \) represents the “local kinetic energy” of the equivalent one-dimensional problem, and illustrates the behaviour of the system for different values of the field strength. Since \( \Phi_+ (y \to \infty) \to -\infty \) for all values of the field strength, the solutions of (16) are always damped at infinity and thus correspond to bound state solutions. Eq. (15), however, can lead to oscillatory solutions at infinity. If we define \( A = \kappa - F \), we are left with a one-dimensional problem with an effective string tension \( A \). The case \( A < 0 \) leads to runaway solutions since it corresponds to a Coulomb problem with an effective deconfining potential.

It is straightforward to calculate the critical field strength \( F_c \), at which tunneling occurs even classically. If we neglect the \( x^{-2} \) term in \( \Phi_- (x) \), which is small in the vicinity of the barrier, we find
\[ F_c = \kappa + \frac{1}{2} (\beta^{(11)} / \beta) \epsilon^2 = \mu^{-2} \beta^{-3} (\sigma + \frac{1}{2} E^2 / \beta^{(11)}) . \]

(19)

In this equation, the binding energy \( E \) and the charge parameter \( \beta^{(11)} \) are determined by solving (15) and (16) semiclassically, i.e. adjusting it such that the WKB quantization conditions are met:
\[ \int_0^\infty \Phi^{1/2} (x; \beta^{(11)}, E) \, dx = \frac{1}{2} \pi \, , \]

(20)
\[ \int_0^\infty \Phi^{1/2} (y; \beta^{(2)}, E) \, dy = \frac{1}{2} \pi \, . \]

(21)

In practice this is done by solving (20) and (21) independently by pairs \( (\beta^{(11)}, E) \) and \( (\beta^{(2)}, E) \) respectively. The energy is then obtained by identifying the point where the trajectories intersect, by requiring \( \beta^{(2)} = 2 \beta - \beta^{(11)} \). A number of intersecting trajectories are shown in fig. 1, for field strengths between \( BF = 0 \) and \( BF = \sigma \). As expected, the energy is lowered by applying stronger fields. This trend is indicated by the dotted curve.

It turns out that for a field which exactly counter-balances the string tension in the “\( \Phi_- \)-sector”, the binding energy becomes exceedingly small. If we use the parameters \( \mu = 1.37 \) GeV, \( \beta = 0.507 \) and \( \sigma = 0.17 \) GeV\(^2\) to fit the 1s, 1p and 2s levels of charmonium, we find \( E = -0.3 \) MeV. This implies that this field is already almost critical. Moreover, the asymmetry in the charge distribution is nearly maximal in this situation. Consequently, a field strength only marginally larger than the string tension disintegrates the bound state completely. Note that for very heavy quark systems the string tension can be neglected, in which case the critical field strength is entirely dictated by the binding energy squared. Since this is much larger for the heavy systems, the critical field is correspondingly larger.

This “sudden death” scenario of \( J/\psi \) disintegration might be modified by a more detailed investigation of the wavefunction \( \psi(\xi, \eta) \) obtained by solving the coupled equations (9) including the cross term. Since a semi-classical analysis is not readily accomplished in this case, it seems appropriate to take the “size” of the \( J/\psi \) in the \( (\xi, \eta) \) plane as a measure of its survival in the strong-field environment. This approach is presently being pursued.

To summarize, we have shown that in a coherent chromo-electric field, \( J/\psi \) suppression is substantial for field strengths that are comparable to the QCD string tension. The suppression (by disintegration)
follows from strong-field ionization and is the analogue of the ionization of hydrogen atoms in strong electric fields. This suppression takes place in a post collision state of minimum entropy and maximum coherence. It does not require any QGP formation. This attitude is of course extreme. However, it clearly illustrates the fact that $J/\psi$ suppression cannot be used as a clear cut signature for QGP formation. Whether the suppression mechanism by field ionization is able to describe the available experimental data has to be determined by a careful analysis of the dependence of the suppression rate on transverse momentum and transverse energy of secondary particles.

References
