SOLITON QUANTIZATION IN CHIRAL MODELS WITH VECTOR MESONS ☆

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A canonical quantization of the skyrmion with ω-mesons is presented using Dirac theory of quantization under constraints. After removing the spurious spin–isospin zero modes from the meson spectrum, an explicit meson–nucleon hamiltonian is constructed. In the strong ω-coupling limit the semi-classical results are recovered. The relevance of this approach to low-energy meson–nucleon physics is stressed.

In the past few years, the Skyrme model has provided important insights to hadronic structure at low energy [1,2]. This non-perturbative approach is believed to relate to QCD in two ways. First, asymptotically in the sense that $N_c \to \infty$ QCD is believed to truncate to an effective theory of weakly coupled mesons that bears some relation to the original Skyrme model. Second, dynamically in the sense that it incorporates the important aspects of current algebra.

The Skyrme model is rooted in the non-linear σ-model, where the elementary fields are pions. The topologically stable soliton configurations inherent in the model yield baryon properties that are in fair agreement (30%) with the QCD baryons. Both scale and counting arguments seem to support the inclusion of vector mesons such as the ω, ρ, A_1 etc., whose masses lie in the 1 GeV-range. If indeed QCD should reduce to an effective mesonic theory at low energy as suggested by the large N_c arguments, then the more realistic the meson theory, the better the baryon properties.

Recently, Adkins and Nappi [3] have suggested to use the isoscalar ω-meson to stabilize the skyrmion. Using semiclassical arguments, they have concluded that the bulk properties of the nucleon and Δ-isobar remained qualitatively similar to the ones derived in the conventional Skyrme model. Similar arguments were then used by several other authors [4–13] to extend the model to the lowest-lying vector mesons (ρ, A_1). The same semiclassical procedure was used with chiral quark models in the presence of vector mesons [14,15].

So far, there have been few systematic approaches to the Skyrme model [16,17] (none to its chiral gauge variants) from the point of view of a relativistic quantum field theory. A direct operator treatment is unavailable. The commonly used approach starts from the classical soliton solutions, and then uses a naive semiclassical quantization of the collective degrees of freedom. These quantization arguments are, however, limited. In order to describe meson–nucleon physics at low-energy using Skyrme-like models, it is imperative to go beyond these semiclassical arguments. Only then, one can extract the pertinent meson–baryon form factors and analyze the relevance of vector dominance in the model. A non-perturbative meson–nucleon hamiltonian would be very useful for a systematic analysis of the NN and N$\bar{\text{N}}$ interaction [18] in the spirit of boson-exchange models, and shed more light on the structure of the isoscalar and isovector meson exchange currents. At this stage, we should point out that the model is not renormalizable. In other words, there is no consistent way of dealing with the infinities that will arise in the quantum approach. We expect, however, that in the soliton sector the skyrmion size will provide natural cutoffs or form factors that would cut down the ultraviolet part of the meson fluctuations in most of the physical observables we are interested in. For in-

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stance, while the zero-point correction to the masses is plagued with perturbative UV divergences, the mass splittings are not and can be calculated in the present context. It is important, however, that the physical quantities extracted from the present approach do not depend on the short distance character of the meson fluctuations.

Having said this, we would like to show in this letter how one can construct a meson–nucleon hamiltonian starting from skyrmions, that explicitly differentiate between the bound states (N, Δ, ...) and the asymptotic mesons (π, ω, ...). In a subsequent letter [19] we shall, as an application of the formalism presented here, calculate the width of the Δ-isobar. A more detailed account of the construction will be given elsewhere [20]. Starting from classical solitons, we will use the canonical version of the collective coordinate method [21] [#1] to quantize the collective degrees of freedom as well as the quantum fluctuations. This method is based on Dirac theory of quantization under constraints [22]. In its canonical form, the procedure avoids ordering ambiguities usually present in the functional approach [23,24].

To illustrate our points we will discuss the ω-stabilized version of the Skyrme model [3]. For simplicity and clarity we will not address the issue of translational invariance and the recoil problem. The generalization of our arguments to more realistic vector models is straightforward though tedious, and will be reported elsewhere [25,20].

We take as dynamical variables the parameters that characterize the isospin position of the classical hedgehog solution together with the quantum fluctuations around it. The presence of spurious zero modes in the small oscillation expansion requires careful quantization conditions. The zero modes arise from the invariance of the classical solution under spin–isospin rotation. By imposing the proper constraints on the various fields to separate the zero modes from the rest of the meson spectrum, we derive an effective hamiltonian that involves explicit baryons (N, Δ) as well as mesons (π, ω). Somewhat surprisingly, we observe that the πN coupling is of order $N_c^{-3/2}$ while the ωN coupling is of order $N_c^{-1/2}$. The absence of a πN coupling to order $N_c^{-1/2}$ is related to our gauge conditions. Since in our case the ω-meson is still present in the baryon–meson hamiltonian, the resulting baryon spectrum is different from the one discussed by Adkins and Nappi. We show explicitly how to recover their spectrum in the strong ω-coupling limit.

[#1] In this approach the issue of hermiticity is subtle. We refer to ref. [17] for a critical discussion.

Consider a non-linear σ-model minimally coupled to a massive spin-1 ω-meson [3],

$$\mathscr{L}=\tfrac{1}{2}\partial_\mu\Phi g(\Phi)\partial_\mu\Phi+f_\pi^2 m_\pi^2[\cos(f_\pi^{-1}\Phi)-1]$$
$$-\tfrac{1}{4}\omega_{\mu\nu}^2+\tfrac{1}{2}m_\omega^2\omega_\mu^2+g_\omega\omega_\mu B_\mu\,, \tag{1}$$

where Φ is the isotriplet pion-field and B_μ the usual topological current. Here g_ω characterizes the strength of the $\omega\to 3\pi$ decay in the vacuum, and $g_{ab}(\Phi)$ is the induced metric on the curved O(3) manifold. As for any Proca field, ω_0 is a constrained variable and not an independent degree of freedom. With this in mind, the classical hamiltonian associated to (1) reads

$$\mathscr{H}=\tfrac{1}{2}(\Pi_\Phi+g_\omega\omega_i\dot{B}_i)g^{-1}(\Phi)(\Pi_\Phi+g_\omega\omega_i\dot{B}_i)$$
$$+\tfrac{1}{2}\nabla_i\Phi g(\Phi)\nabla_i\Phi+f_\pi^2 m_\pi^2[1-\cos(f_\pi^{-1}\Phi)]$$
$$+\tfrac{1}{2}\Omega_j^2+\tfrac{1}{4}\omega_{ij}^2+\tfrac{1}{2}m_\omega^2\omega_j^2+\frac{1}{2m_\omega^2}(\partial_j\Omega_j-g_\omega B_0)^2\,, \tag{2}$$

where Π_Φ and $\boldsymbol{\Omega}$ are the momenta canonically conjugate to Φ and $\boldsymbol{\omega}$ respectively. They satisfy canonical Poisson brackets. If $A[\Phi]$ is a functional of Φ, then we define $A'=\delta A/\delta\Phi$ and $\dot{A}=\delta A/\delta\dot{\Phi}$. Note that as expected, $\mathscr{H}$ is positive and thus bounded from below. The static soliton configurations are solution to the time-independent Hamilton equations. The static hedgehog solution is given by

$$\boldsymbol{\phi}(\boldsymbol{x})=f_\pi\hat{\boldsymbol{r}}F(r),\quad \boldsymbol{\omega}=0 \tag{3}$$

with $F(r)$ subject to the boundary conditions $F(0)=\pi$ and $F(\infty)=0$. Contrary to recent claims, this solution is stable against homogeneous scale transformations. We stress again that ω_0 is not an independent field, and its scaling properties follow from the constraint equation $\partial\mathscr{L}/\partial\omega_0=0$. In the discussion of ref. [26] this point has been ignored.

Global rotations, isorotations and translations of (3) yield configurations with the same energy, suggesting that the vacuum is infinitely degenerate. Naive quantization around the classical configuration (3) using the small oscillation expansion is doomed because of the presence of zero-energy modes. These are static solutions to the small oscillation problem that immediately result from the broken invariances. Because of the hedgehog character of (3), spin zero modes are identical to isospin zero mode and are given by (unnormalized)

$$\phi_0^a(\boldsymbol{x}) = T^a \phi(\boldsymbol{x}), \quad \omega = 0 . \tag{4}$$

The T's are the usual O(3) generators in the adjoint representation. Note that these zero modes are also solution to the differential equations resulting from $[H, \Phi] = 0$ by variation. That this is always the case follows of course from spin–isospin invariance. To treat these modes properly, we will use the collective coordinate method [21,27,28]. For that, define

$$\Phi^a(x) = R^{ab}[\theta]\,[\phi^b(\boldsymbol{x}) + \xi^b(\boldsymbol{x}, t)] , \tag{5}$$

where R is an element of the symmetry group of the vacuum, and $\xi(x)$ the body-fixed pion field. Since the $\boldsymbol{\omega}$ field is an isosinglet that vanishes in the classical vacuum, its status remains unaffected by a rigid isorotation R.

The physical isospin I is the generator of left transformations $R \to (1 + \mathrm{i}\epsilon^a T^a)R$, whereas the intrinsic isospin J generates right transformations $R \to R(1 + \mathrm{i}\epsilon^a T^a)$. Owing to the hedgehog character of (3), the latter may be identified with the physical spin to lowest order. Since (5) is invariant under the "gauge transformation"

$$R \to R(1 + \mathrm{i}\epsilon^a T^a) ,$$
$$\phi + \xi \to (1 - \mathrm{i}\epsilon^a T^a)(\phi + \xi) . \tag{6}$$

Canonical quantization of (1) now yields first class constraints of the form

$$G_a \equiv J_a + \mathrm{i} \int \mathrm{d}^3x\, \Pi T_a(\phi + \xi) = 0 , \tag{7}$$

where Π is the momentum conjugate to ξ. Thus, the structure of these constraints is unchanged by the interaction. To eliminate the "gauge" redundancy we choose the subsidiary constraint

$$\chi_a \equiv \int \mathrm{d}^3x\, \xi g(\phi) T^a \phi = 0 , \tag{8}$$

as suggested by a straightforward Taylor expansion of (5). The Poisson bracket of (7) and (8) is given by

$$\Lambda_{ab} = [G_a, \chi_b] = \int \mathrm{d}^3x\, T_a(\phi + \xi) g(\phi) T_b \phi . \tag{9}$$

To lowest order Λ_{ab} is equal to $\Lambda_\pi \delta_{ab}$, where Λ_π is the classical moment of inertia given by the norm of the zero modes (4). In so far there is no contribution to the moment of inertia from the isoscalar ω-field. Since (9) is non-vanishing, the constraints are now second class, and may be taken as strong operator identities. Correspondingly, the commutators are also changed from Poisson brackets to Dirac brackets.

In order to disentangle the collective degrees of freedom from the fluctuations, we further choose to decompose $\Pi = \Pi_\mathrm{L} + \Pi_\mathrm{T}$ such that Π_L is proportional to the zero mode, and [#2]

$$\int \mathrm{d}^3x\, \Pi_\mathrm{T} T_a \phi = -g_\omega \int \mathrm{d}^3x\, \omega_i \dot{B}_i T_a \phi \tag{10}$$

as suggested by the time derivative of (8). Solving for Π_L in (7) gives

$$\Pi_\mathrm{L} = [-\mathrm{i}g(\phi) T_a \phi] \Lambda_{ab}^{-1} \times \left(J_b + \mathrm{i} \int \Pi_\mathrm{T} T_b \xi + \mathrm{i} g_\omega \int \omega_j \dot{B}_j T_b \phi \right), \tag{11}$$

where Λ_{ab} is the tensor of inertia defined in (9). Now (Π_T, ξ) and (Ω_k, ω_k) can be viewed as conjugate variables that satisfy Dirac and ordinary brackets respectively. With our choice of (Π_T, ξ) it is easy to show that Φ and Π_Φ satisfy canonical commutation relations, as of course expected.

Recalling that $\dot{R}$ scales like N_c^{-1}, and ξ, $\boldsymbol{\omega}$ like $N_\mathrm{c}^{-1/2}$, then to order N_c^{-1} the normal ordered hamiltonian reads (in the absence of recoil)

$$H = M_\mathrm{S} + H_{\xi\omega} + g_\omega \omega_i \dot{B}_i (-\mathrm{i} T_a \phi) \frac{J_a}{\Lambda_\pi} + \frac{J^2}{2\Lambda_\pi} , \tag{12}$$

where $H_{\xi\omega}$ is the order 1 meson hamiltonian in the hedgehog background. The time-independent spin–isospin zero modes (4) are solution to the background field equations associated to $H_{\xi\omega}$. They are,

[#2] There is of course some arbitrariness in choosing these gauge conditions. This arbitrariness, however, is severely limited by requiring canonical commutation relations for the original fields throughout. Physical quantities are, of course, independent of this choice of gauge.

however, excluded from the meson spectrum because of (8). Note that the ω–nucleon coupling is of order $N_c^{-1/2}$. To this order, the pion–nucleon coupling vanishes because of the gauge condition (10). It shows up to order $N_c^{-3/2}$ in the form

$$H_{\xi N} = -\frac{J^a}{2\Lambda_\pi}\left[\tfrac{1}{2}T^a\phi g'(\phi)\cdot\xi T^b\phi + T^a\phi g(\phi)T^b\xi\right]\frac{J^b}{\Lambda_\pi} + \text{h.c.} \tag{13}$$

and vanishes for a constant pion field (Dashen point). The form of this coupling is very similar to the one used in the so-called static Chew model [29]. In fact such a coupling was discussed long ago by Pauli and Dancoff [30] in the context of the strong coupling approximation. At this stage we should point out that even though (13) scales like $N_c^{-3/2}$, this does not imply that the pion–nucleon coupling constant scales like $N_c^{-3/2}$. As a matter of fact, (13) yields a pion–nucleon coupling constant that is exactly the one obtained from conventional source theory [20]. To see this, we have to rewrite (13) in terms of the "extrinsic" pion field $\eta \to R\xi$. Since asymptotically the time component of the axial current reads $A_0 = R(\Pi_T + \Pi_L)$, we can rewrite the pion–nucleon interaction (13) in the following form:

$$H_{\pi N} = \frac{1}{2f_\pi}\int d^3x\, \eta \dot{R}R^{-1}A_0\,. \tag{14}$$

To leading order in N_c (14) simplifies into

$$H_{\pi N} = \frac{1}{2f_\pi}\int d^3x\, \eta\partial_0 A_0^{(0)} + \text{h.c.}\,, \tag{15}$$

which is the time component of the expected pion–nucleon coupling term. Remember that to leading order in N_c the spatial part of the coupling term vanishes in the massless limit ($\nabla \mathbf{A}^{(0)} = 0$). (15) when sandwiched between the proper pion–nucleon states, yields the ordinary pion–nucleon coupling constant. A more detailed account and analysis of the above hamiltonian will be given elsewhere [20]. At this stage we should mention that recently, Schnitzer [31] has proposed a pion–nucleon hamiltonian for the original Skyrme model. However, in his treatment of the pion fluctuations both the constraint conditions and the zero-mode problem have been ignored.

If we were to treat the ω–nucleon coupling in (12) perturbatively, then the baryon spectrum is given by (ignoring zero point fluctuations)

$$H_B = M_S + \frac{J^2}{2\Lambda_\pi}\,. \tag{16}$$

In the case where the ω coupling in (12) is strong (being of order $N_c^{-1/2}$ it can involve an energy shift of order N_c^{-1}), it can affect quantitatively the baryon spectrum as described by (16). Indeed, from the general results of ref. [20], the equation of motion for the ω field reads (ignoring local background effects)

$$(\Box + m_\omega^2)\omega_j + g_\omega^2 \frac{\dot{B}_j(-iT_a\phi)}{\Lambda_\pi}\int d^3x\, \omega_j \dot{B}_j(-iT_a\phi) = -g_\omega \dot{B}_j(-iT_a\phi)\frac{J_a}{\Lambda_\pi}\,. \tag{17}$$

The non-local term in (17) is left over by the constraint condition (10). This integro-differential equation can be integrated at once to give

$$\omega_j = \omega_j^{\pm} + (\nabla^2 - m_\omega^2)^{-1} g_\omega \dot{B}_j(-iT_a\phi)\frac{J_a}{\Lambda_\pi + \Lambda_\omega} \tag{18}$$

with

$$\Lambda_\omega = \tfrac{1}{3}g_\omega^2 \int d^3x\, \dot{B}_i(-iT_a\phi)(-\nabla^2 + m_\omega^2)^{-1} \times \dot{B}_i(-iT_a\phi)\,. \tag{19}$$

and where $\omega^{\pm}$ are free massive waves subject to the usual causal boundary conditions. This yields a baryon spectrum of the form

$$H_B^* = M_S + \frac{J^2}{2\Lambda_\pi}\left(1 - \frac{\Lambda_\omega}{\Lambda_\pi + \Lambda_\omega}\right)\,. \tag{20}$$

There are no other meson-induced effects on the spectrum to order N_c^{-1}. The ω induced term in (20) is attractive ($\Lambda_\omega > 0$) since it is driven by the space components of the vector interaction. Remember that the time component of the vector interaction is repulsive and provides the skyrmion with the necessary repulsion at short distances to balance the overall long-range pion attraction. The higher the spin the larger the down-shift. The effective moment of inertia is $\Lambda = \Lambda_\pi + \Lambda_\omega$, and agrees with the one originally derived by Adkins and Nappi [3].

To summarize, for a weak ω coupling, the moment of inertia is exclusively pionic and given by Λ_π, while for a strong ω coupling it is given by Λ. The semiclassical quantization arguments presented in ref. [3] rely on the use of the equations of motion for the ω field in the lagrangian, prior to the construction of the hamiltonian. This procedure is in general not correct. However, since the ω field in (1) enters at most quadratically, then in a functional formulation it can be eliminated from the starting action using the conventional Hubbard or gaussian transformation, thus amending in a way the Adkins and Nappi procedure in the semiclassical limit. Their procedure, however, was never meant to be more general than that. The variational arguments used in ref. [3–15] do not follow from conventional quantization schemes, and can not be immediately supported by the canonical procedure discussed above. The latter allows for a systematic analysis of meson–nucleon physics at low energy.

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